Entangled Neural Networks

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Abstract. Entangled Neural Networks (ENNs) are proposed on the basis of quantum teleportation and its extension to intelligent sense. Following a general hypothesis that conscious decisions are stimulated by the influence of a lot of unconscious factors, ENNs show the potential for inference and manipulation of a huge amount of knowledge using quantum parallelism.

1 Introduction

Quantum teleportation is surprising the scientific community since it can transmit an unknown quantum state following the no cloning principle of quantum mechanics [2,4]. During teleportation, some limited information is transmitted in the classical channel and the quantum state is transmitted in the quantum channel. Even though the original quantum state can be reconstructed (with the price of destroying the original state), nothing is learned during the transmission about the original state [17]. Considering the powerful learning ability of Artificial Neural Networks (ANN) [10,11], Entangled Neural Networks (ENNs) are proposed in this sense to learn information from inside and outside the system and to infer and manipulate the huge amount of knowledge using quantum parallelism. There is some interesting literature about the use of ANN with quantum computing [6,14,16,1,19,25,17,20,23, 26], and especially, a review from Hirsh et al. [12,13,15,24]. More recent paper [3] about non-local quantum evolution of entangled ensemble states in neural nets shows interesting discussion in this line.

Neuron *A* (Alice), neuron *B* (Bob), an EPR source [2] and some connections (classical and quantum channels) form a basic ENN (or unit). The operation of every ENN looks like quantum teleportation but the measurement of neuron *A* is orientated with intelligence. Connections in certain manners among these units construct ENNs. With this arrangement, the simulation of whole ENNs tries to follow a general hypothesis in which the conscious decision is stimulated by the influences of a lot of unconscious factors [18,22]. In ENNs, there is no repeated learning sequence as is the case with classical ANN, therefore, the decoherence problem may be reduced. We briefly describe the basic concepts of quantum teleportation and the development of ENNs. Through an example of temperature adjusting, we show the application of Hebb´s learning law and the decision sequence using ENNs.

2 Brief Review of Quantum Teleportation

In 1993, Bennett et al. developed quantum teleportation to transmit an unknown quantum state of a particle using both classical and quantum channels [2]. This technique is based on the EPR paradox, Bell´s theory and other quantum physics theories and experiments [4,7]. The following is a brief description of quantum computing and quantum teleportation based on Rieffel and Polak´s note [21].

In quantum mechanics, quantum state spaces and the transformations acting on them can be described in terms of vectors and matrices or in the more compact bra/ket notation which was invented by Dirac. Kets like $|x\rangle$ denote column vectors and are typically used to describe quantum states. The matching bra, $\langle x|$, denotes the conjugate transpose of $|x\rangle$. A quantum bit, or qubit, is a unit vector in a two dimensional complex vector space for which a particular basis, denoted by $\{|0\rangle, |1\rangle\}$. Unlike classical bits, qubits can be in a superposition of $|0\rangle$ and $|1\rangle$. A superposition is closely related to the familiar mathematical principle of linear combination of vectors. Quantum systems are described by a wave function that exists in a Hilbert space. In case of $a|0\rangle + b|1\rangle$, a and b are complex numbers such that $|a|^2 + |b|^2 = 1$. If such a superposition is measured with respect to the basis $\{|0\rangle, |1\rangle\}$, the probability is $|a|^2$ when the measured value is $|0\rangle$ and $|b|^2$ when $|1\rangle$.

Entanglement is the potential for quantum states to exhibit correlations the cannot be accounted for classically. These correlations somehow exist in a superposition as well. When the superposition is destroyed, the proper correlation is somehow communicated between the qubits, and it is this communication that is the crux of the entanglement. For example, the state $1/\sqrt{2}\,(|00\rangle+|11\rangle)$ is maximally entangled. A source that generates these two maximally entangled particles, is called an EPR pair, where EPR is from the names of Einstein, Podolsky and Rosen who proposed a gadanken experiment using entangled particles in a manner that seemed to violate fundamental principles relativity. Reader can get more information from [2]. The followings are the main steps of a tele-portation procedure.

1. Alice (sender), Bob (receiver), an EPR source, a classical channel and a quantum channel constitute a teleportation system (Figure 1).
2. Alice and Bob wish to communicate. Each is sent one of the entangled particles from the EPR source making up an EPR pair: $\varphi_0 = 1/\sqrt{2}\,(|00\rangle + |11\rangle)$.
3. Alice wants to send the state of the qubit $\phi = a|0\rangle + b|1\rangle$ to Bob through a classical and a quantum channel. She applies the decoding to the qubit ϕ and her half of the entangled pair. The starting state is quantum state

$$\begin{aligned} \phi \otimes \varphi_0 &= 1/\sqrt{2}\,(a|0\rangle \otimes (|00\rangle + |11\rangle) + b|1\rangle \otimes (|00\rangle + |11\rangle) \\ &= 1/\sqrt{2}\,(a|000\rangle + a|011\rangle + b|100\rangle + b|111\rangle) \end{aligned} \tag{2.1}$$

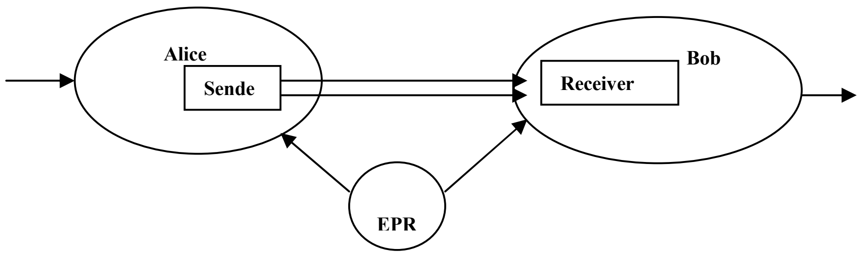

Fig. 1. Quantum teleportation system

4. Alice now applies $C_{not} \otimes I$ and $H \otimes I \otimes I$ to this state, where C_{not} is controlled-NOT gate; I is the identification transform; H is the Hadamard transform[21].

$$
\begin{aligned}
&(H \otimes I \otimes I) (C_{not} \otimes I) (\phi \otimes \varphi_0) \\
&\quad = (H \otimes I \otimes I) (C_{not} \otimes I) \, 1/\sqrt{2} \, (a|000> + a|011> + b|100> + b|111>) \\
&\quad = (H \otimes I \otimes I) \, 1/\sqrt{2} \, (a|000> + a|011> + b|110> + b|101>) \\
&\quad = \tfrac{1}{2} \, (|00>(a|0> + b|1>) + |01>(a|1> + \\
&\qquad b|0>) + |10>(a|0> - b|1>) + |11>(a|1> - b|0>))
\end{aligned} \tag{2.2}
$$

5. Alice controls the first two bits and Bob controls the last one. She measures the first two qubits to get one of |00>, |01>, |10> and |11> with equal probability. She sends the result of her measurement as two classical bits to Bob.
6. Depending on Alice´s measurement, the quantum state of Bob´s qubit is projected to (a|0> + b|1>), (a|1> + b|0>), (a|0> - b|1>) or (a|1> - b|0>) respectively. These are the combination of the basic states (0 and 1) and the phase states (+ and -).
7. Bob receives the two classical bits from Alice. He know how to decode his half of the entangled pair to the original state of Alice´s qubit using an appropriate decoding transformation. Table 1 shows the conditions for which particular transformations will be selected [21], where,

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \; X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix};$$

$$Y = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}; \; Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Table 1 State Transformation of Qubit by Bob

Bits received	State	Decoding transformation
00	a|0> + b|1>	I (the identification transformation)
01	a|1> + b|0>	X (the negation transformation)
10	a|0> - b|1>	Z (the phase shift transformation)
11	a|1> - b|0>	Y ($Y = ZX$)

3 Construction of Entangled Neural Networks

3.1 Basic unit of ENNs: an ENN

In order to develop an intelligent system using the vantages of quantum teleportation, some steps and elements are modified to form a basic unit of ENNs, an ENN. Suppose, there are three units: unit I, II, III. The following operations are within Unit II if we do not explicitly mention the unit number.

1. Neuron *A* (Alice, sender), neuron *B* (Bob, receiver), an EPR source, some connections (at least one classical and one quantum channel) constitute a basic unit (an ENN) (Figure 2).

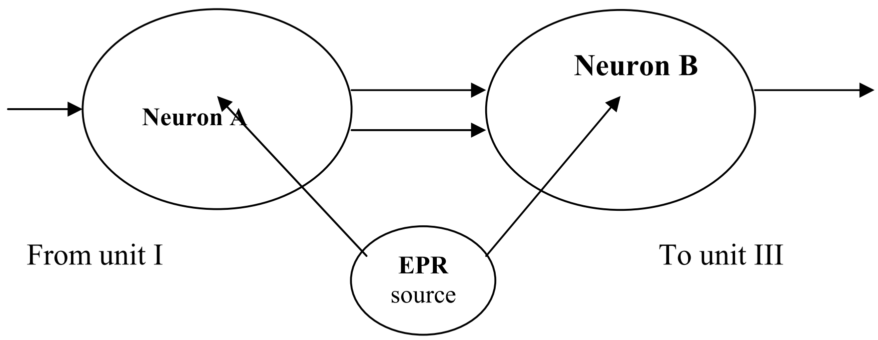

Fig. 2. Unit II of ENNs

2. Neuron *A* and neuron *B* are each sent one of the entangled particles from the EPR source making up an EPR pair: $\varphi_0 = 1/\sqrt{2}\ (|00> + |11>)$.
3. In neuron *A*, we define the state of a qubit $\phi = a|0> + b|1>$ to represent some decision factors. For example: the temperature in the room is high with a probability of $|a|^2$ or low with a probability of $|b|^2$.
4. Neuron *A* receives information from another unit, e.g. Unit I through a connection (classical or quantum channel) and wants to transmit the qubit ϕ to neuron *B* through the connections between them (classical and quantum channels). *A* applies the decoding to ϕ and its half of the entangled pair. The starting state is the quantum state:

$$\phi \otimes \varphi_0 = 1/\sqrt{2}\ (a|0> \otimes (|00> + |11>) + b|1> \otimes (|00> + |11>)$$
$$= 1/\sqrt{2}\ (a|000> + a|011> + b|100> + b|111>) \quad (3.1)$$

5. Neuron *A* now applies $C_{not} \otimes I$ and $H \otimes I \otimes I$ to this state, where C_{not} is controlled-NOT gate; *I* is the identification transform; *H* is the Hadamard transform.

$$(H \otimes I \otimes I)(C_{not} \otimes I)(\phi \otimes \varphi_0)$$
$$= (H \otimes I \otimes I)(C_{not} \otimes I)\ 1/\sqrt{2}\ (a|000> + a|011> + b|100> + b|111>)$$
$$= (H \otimes I \otimes I)\ 1/\sqrt{2}\ (a|000> + a|011> + b|110> + b|101>)$$

$$= ½ (|00>(a|0> + b|1>) + |01>(a|1> + b|0>) + |10>(a|0> - b|1>) + |11>(a|1> - b|0>)) \quad (3.2)$$

6. In neuron *A*, we can measure the first two bits and in neuron *B*, the last one. Due to the information from outside and inside of the unit, this is defined as a *decision key*, τ. Using τ we can measure the first two qubits to get one of |00>, |01>, |10> and |11> with a special probability. The measurement result is defined as a *measurement key*, ט. Using Grover's algorithm [9], the probability according to |τ> is amplified as $|p|^2$ and probability of others is reduced as $|q|^2$, where $|p|^2 + 3|q|^2 = 1$ and $|p|^2 >> |q|^2$. Then, υ (equals τ with a probability of $|p|^2$) with two classical bits is sent to neuron *B*. This step is very different from the teleportation procedure. Section 4 will discuss how to get τ from external (i.e. from other units) and internal (from neuron *A*) knowledge, and how to use υ to get the measurement from neuron *B*. Let Grover() be the subroutine of the Grover algorithm. In case of $\tau = 01$, there is:

$$\text{Grover}((H \otimes I \otimes I) (C_{not} \otimes I) (\phi \otimes \varphi_0)) = q\,|00>(a|0> + b|1>) + p\,|01>(a|1> + b|0>) + q\,|10>(a|0> - b|1>) + q\,|11>(a|1> - b|0>) \quad (3.3)$$

7. Depending on neuron *A*´s measurement (υ), the quantum state of neuron *B*´s qubit is projected to (a|0> + b|1>), (a|1> + b|0>), (a|0> - b|1>) or (a|1> - b|0>) respectively. They are combinations of the amplitude states (0 and 1) and the phase states (+ and -).
8. When neuron *B* receives the two classical bits (υ) from neuron *A*, it may measure one of four states as shown in Table 2.
9. The state of the qubit in neuron *B* in Table 2 can be measured and the result is transmitted to other units, such as Unit III through their connections.

Table 2 State Combinations of the Qubit of Neuron *B*

Measurement key (υ)	State	Measurement key (υ)	State
00	a|0> + b|1>	10	a|0> - b|1>
01	a|1> + b|0>	11	a|1> - b|0>

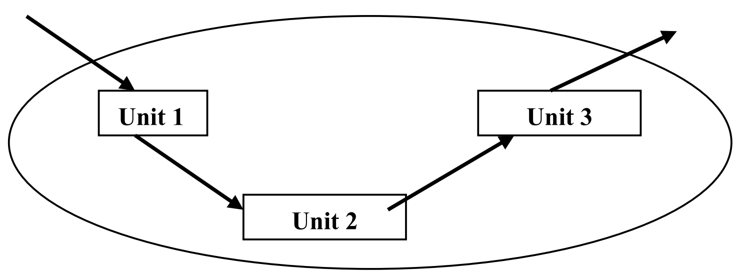

Fig. 3. Simple ENNs with three basic units

3.2 Structure of ENNs

Some basic units form ENNs. In Figure 3, three units are connected. Unit I receives an input and Unit III sends an output.

4 Temperature Adjusting Problem and ENNs Resolution

4.1 Temperature adjusting problem

In Summer, the temperature is generally high so it is likely that someone will want to adjust the temperature in his room. The problem space is defined from some related factors. We just select 6 items to show the application of ENNs.

Human factors:

1. Ages(Young, Old);
2. Manner of life(Hedonist, Ascetic);
3. Health(Healthy, Sick);
4. Manner with money(Extravagant, Thrifty);

Natural factors:

5. Temperature(Low, High);

Marketing factors:

6. Cost of adjusting heat (Expensive, Cheap),

An expected decision result will be one of:

1. Cost_Temperature(Expensive, Low); i.e., it costs more money to get a low temperature;
2. Cost_Temperature (Cheap, Low); i.e., it costs less money to get a low temperature;
3. Cost_Temperature (Expensive, High); i.e., it costs more money to get a high temperature;
4. Cost_Temperature (Cheap; High); i.e., it costs less money to get a high temperature;

We use ENNs to make a decision for a thrifty and old gentlemen to get a low temperature. So, Thrifty, Old and Low are defined as the *requirement factors*. Each of these factors will be represented as a phase state in its unit. Three basic units are needed to form ENNs as shown in Figure 3.

4.2 Quantum state definition and decision result measurement

As described in section 3.1, the qubit ϕ will be teleported in a unit. It is represented in two parts: *amplitude state* and *phase state* according to the problem as in Table 3. The amplitude state is used to form the *influence factors*. The phase state forms the *requirement factors*.

Table 3 State Representation for the Qubit ϕ

Unit	Phase (requirement factors)		Amplitude (influence factors)	
	+	-	1	0
Unit I	Young	Old	Hedonist	Ascetic
Unit II	Extravagant	Thrifty	Healthy	Sick
Unit III	Low	High	Expensive	Cheap

For Unit I, the quantum state of neuron *A*´s qubit is defined as ϕ = a| Ascetic > + b| Hedonist >. This means that the people in the room is an *Ascetic* with a probability of $|a|^2$ or a *Hedonist* with a probability of $|b|^2$. The quantum state of neuron *B*´s qubit will be projected to (a|0> + b|1>), (a|1> + b|0>), (a|0> - b|1>) or (a|1> - b|0>) respectively depending on the *measurement key* υ. These four possible states are defined as in Table 4. The possible measurements of the state are also the possible decision results from Unit I. This result will be passed to Unit II. We can also list the possible measurements of the qubit in neuron *B* (possible decision results) for Units II and III.

Table 4 Possible Measurements of Neuron *B*, i. e. Decision Result from Unit I

Measurement key (υ)	State	Measurement (Manner of life_Ages)	
		0 (probability = $|a|^2$)	**1** (probability = $|b|^2$)
00	a|0> + b|1>	(Ascetic, Young)	(Hedonist, Young)
01	a|1> + b|0>	(Hedonist, Young)	(Ascetic, Young)
10	a|0> - b|1>	(Ascetic, Old)	(Hedonist, Old)
11	a|1> - b|0>	(Hedonism, Old)	(Asceticism, Old)

4.3 Learning from Inside and Outside of a Unit (how to get τ)

The decision key τ is determined from two kinds of information from inside and outside of a unit, it consists of two numbers. The first part is 0 or 1 which represents information about the proper phase state of the *requirement factor* in the unit itself and the second part is also 0 or 1 which represents the information learned from outside the unit. For the representation of qubit ϕ, we also need to represent the quantum state of the qubits φ_0 from EPR source. This is the same thing as Table 3 but the amplitude and phase states all use 0 and 1 (in Table 3, 1 and 0 are used for amplitude, + and – for phase). Note the difference between Tables 3 and 5.

Table 5 State Representation for the Qubits φ_0

Unit	Phase		Amplitude	
	0	**1**	**0**	**1**
Outside of ENN	Low	High	-	-
Unit I	Young	Old	Hedonist	Ascetic
Unit II	Extravagant	Thrifty	Healthy	Sick
Unit III	Low	High	Expensive	Cheap

How can we transfer this information to every unit as useful knowledge? The following steps are needed to obtain the decision key τ (*i*), where *i* means the Unit I, II or III. We also use the same notation for the measurement key υ, i.e., υ(i).

1. *Using the state of the requirement factor to define the first part of $\tau(1)$.* For Unit I from Table 3, the *requirement factor* is Old which is represented as 1 in Table 5. So, $\tau(1) = 1y$, where y is the second part of τ which will be defined by outside information to Unit I.
2. *Using the information from the outside of the unit to determine the second part of $\tau(1)$.* In case of Unit I, neuron A learns information from outside the ENNs: we need a low temperature. From Table 5, we know 0 is used to represent Low. So, $y = 0$, $\tau(1) = 10$.
3. *Using the measurement result to get the decision result of unit I.* Then the measurement key $\upsilon(1)$ equals $\tau(1)$ with a probability $|p|^2$ (equation 3.3) depending the measurement in neuron A. If we really measure that $\upsilon(1) = \tau(1) = 10$, the qubit in neuron B will be projected in the state of a|1> - b|0>, i.e. a|(Ascetic, Old)> - b|(Hedonist, Old)>. We can get a decision result from Unit I as (Ascetic, Old) with a probability of $|a|^2$ and (Hedonist, Old) with a probability of $|b|^2$.
4. *Using the decision result of Unit I to define the second part of decision key $\tau(2)$ of Unit II.* Suppose the measurement result is (Hedonist, Old). Using Hebb´s learning law [Hec90, Hey94], the selection of the degree of the amplitude between both connected units has the same tendency. In our case, we use the *Hedonist* state to orientate Unit II to measure the state of *Healthy*, rather than *Sick*. The decision key for Unit II $\tau(2) = x0$, where x will be defined by the requirement factor of Unit II. Using 0 to represent *Hedonist* comes from Table 5.
5. *Using the state of the requirement factor to define the first part of* $\tau(2)$. For Unit II, and again from Table 3, the *requirement factor* is Thrifty which is represented as a 1 in Table 5. We get $\tau(2) = 10$.
6. *Repeat 3 –5 steps until the last unit of ENNs.*

One result of our simulation of the temperature adjusting problem is: Cost_Temperature (Cheap, Low). This means that the gentlemen prefers to spend less money to get a low temperature.

5 Remarks

The proposal is just a suggestion of one possibility for using quantum computing for ANN application. The basic idea of ENNs is still in its infancy and it wil be necessary to study other related topics more deeply. First of all, we need to study the possibility of considering a teleportation system as an ENN, in both the physics and engineering sense. The following discussions show more properties of ENNs and suggest directions for future research.

1. *Non-repeated learning manner.* In ENNs, a quantum computation only takes place in one unit (an ENN), after the measurement of the reconstructed state in neuron B, the information is transmitted to the next unit. This process avoids the repeated learning strategy of classical ANN and reduces the possible decoherence problem.

2. *Learning and learning law in ENNs.* The learning activity is realized through the *decision key* and *measurement key* of every unit. It is very important to orientate the quantum state to collapse in an intelligent manner, i.e. to get an intelligent decision. We use Hebb´s learning law in our study. More “quantum suitable” learning laws should be discovered as a result of future research.
3. *Possible state generation and combination*. In our study, temperature adjusting problem, six factors are used to form the possibility states. For quantum parallelism it will be necessary to establish how to generate a vast quantity of states and find the best match in the ocean of knowledge. This the most important aspect to follow for our conscious decision hypothesis.
4. *Multi-party communication.* We are only concerned with the teleportation case, i.e. two-party communication. For generalizing ENNs, multi-party communication should be considered [Buh97].
5. *ENNs simulation*. A ENNs simulator should be developed for classical computers to verify the efficiency and the correctness of this new model.

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