In functional programs the order of evaluation need not be fully specified by the programmer, which gives to compilers some freedom in scheduling computations. However, the many possibilities may allow substantial variations in the execution time of a program. So-called “optimal reductions” are rewriting sequences that produce the result of a program using a minimal number of steps, and characterize lower bounds on the evaluation of programs.

This mini-course is a broad introduction to optimality and to efficient reduction strategies in the lambda-calculus and in rewriting theory, ranging from decidability issues to implementation techniques. Some highlights will be:

- the impossibility of defining an effective optimal reduction strategy for the usual lambda-calculus,
- the replacement of terms by graphs to circumvent this problem,
- the decoration of terms with labels to represent graphs without ever manipulating them directly,
- the transformation of programs by lambda-lifting to reduce a higher-order problem to a much simpler first-order problem,
- the relative optimality of a reduction strategy based on lazy evaluation and on memorization of intermediate results in the style of Haskell.

Some references:

Wadsworth, 1971: Semantics and Pragmatics of the Lambda Calculus. [Chapter 4: first historical use of graphs for the evaluation of lambda-terms.]
Barendregt, Bergstra, Klop, Volken, 1976: Some Notes on Lambda Reduction. [Ineffectivity of the optimal strategies.]
Lévy, 1980: Optimal Reductions in the Lambda-Calculus. [The foundations of optimality theory.]
Peyton-Jones, 1987: The Implementation of Functional Programming Languages. [Laziness, graphs, lambda-lifting.]
Lamping, 1990: An Algorithm for Optimal Lambda Calculus Reduction. [First implementation of Lévy’s optimality theory.]
Maranget, 1991: Optimal Derivations in Weak Lambda-Calculi and in Orthogonal Terms Rewriting Systems. [Optimality in first-order systems, and first use of labels to represent graphs.]
Glauert, Khasidashvili, 1996: Relative Normalization in Deterministic Residual Structures. [Abstract account of Lévy’s theory.]
Blanc, Lévy, Maranget, 2007: Sharing in the Weak Lambda-Calculus. [Labels in the lambda-calculus.]
Balabonski, 2012: Axiomatic Sharing-via-Labelling. [Labels in any rewriting system.]